VŠB TECHNICKÁ UNIVERZITA OSTRAVA

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Úvod do funkcionálního programování

behalek.cs.vsb.cz/wiki/Functional_Programming

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Funkcionální styl programování

- Spolus s logickým programováním reprezentuje deklarativní styl programování.
- *Nebudeme-li zacházet do detailů,* Deklarativní styl programování je protiklad k imperativnímu stylu programování.
- Imperativní programování
  - Program je sekvence příkazů.
  - Jednoznačně definováno co se má provádět.
  - Příkazy mají vedlejší efekty (`a=5;`).
  - *Věci* z matematiky mají jiný význam.
- Funkcionální programování
  - Program se skládá z množiny funkcí, které definují co *je*.
  - Vyhodnocení programu je pak vyhodnocení hlavního výrazu.
  - Žádný implicitní stav, funkce nemají vedlejší efekty → referenční transparentnost *(tu chceme...:-)*
  - Blíže k matematice - například pojem proměnné má pořád matematický význam.

- Dobré
  - Výborný mechanismus abstrakce (funkce vyššího řádu, kompozice funkcí).
    - Elegantní a výstižný programy → kratší než imperativní ekvivalent, odolnější vůči chybám → snadnější na údržbu
  - Nové možnosti jako líné vyhodnocování → umožňuje práci s nekonečnými strukturami
    - Velmi dobrý mechanismus jak zpracovat velká data (*big data*).
  - Referenční transparentnost umožňuje kompilátoru (a jiným nástrojům) usuzovat o chování programu a dokonce dokazovat jeho vlastnosti.
  - Absence vedlejších efektů → efektivní a jednoduchá (automatická) parelelizace programu.
- Špatné
  - Ladění - složitější (pokud je program typově korektní, je obtížnější udělat netriviální chybu).
  - Výkon
    - Omezen aktuálním hardwarem - více vyhovuje imperativním jazykům.
    - Abychom dostali *rozumný* výkon, musíme často něco obětovat (OCalm - imperativní konstrukce, nemá líné vyhodnocování) a/nebo provádět komplexní optimalizace.

- Ošklivé - Jak často se ~~čistě~~ funkcionální jazyky používají v reálných aplikacích?
  - Některé jazyky byla úspěšné v určitých oblastech jako Erlang - Elixier pro systémy pracující v reálném čase.
  - My budeme používat **Haskell** (20. (nebo tak nějak) nejpoužívanější pro open-source projekty, anti-span filter Facebooku).
  - Populární programovací jazyky implementují více paradigmat programování, v některých je funkcionální paradigma dominantní (Python, Javascript, C#), některé části ještě vice realizují podstatu čistě funkcionálního programování (LINQ v C#).
  - U některých komponent není explicitně vidět, v jakém jazyce byly implementované (Scala beží na JVM, Elm se kompiluje do jazyka Javascript).

- Pořád ale platí, že funkcionální **styl** programování je často poučíván i v imperativních jazycích.

Haskell

- Tak proč se učíme zrovna Haskell?
- Čistě funkcionální jazyk → ukazuje základní principy
  - Je *jednodušší* než různé dialecty ML, Lisp, ...
- Vyspělý jazyk + množství nástrojů
- Ve stovnání s jinými (imperativními) jazyky **velmi** jednoduchý.
- Obtíže učení se nového programovacího jazyka spočívají v:
  - naučit se syntaxi a sémantiku pro konstrukce z jazyka → *jednoduché* pro Haskell
  - naučit se řešit problém *funkcionálním způsobem* → to bude **hlavní náplní** tohoto kurzu
  - naučit se používat API → zabere obvykle nejvice času, my budeme používat jen malou část API, navíc většinu si *napíšeme* sami

Funkce, kamkoliv se podíváš I

- Funkce
  - Na základě vstupních parametrů vrátí výstupní hodnotu (hodnoty).
  - Definice:

```
name :: Type
name = expression
```

  - Příklady:

```
doubleMe x = x * x
plus x y  = x + y

max x y | x > y = x
        | x <= y = y

factorial 0 = 1
factorial n = n * factorial (n-1)
```

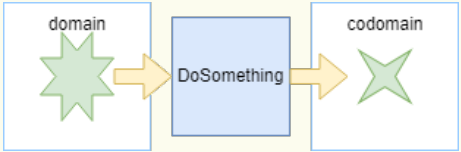
Funkce, kamkoliv se podíváš II

- Proces, kdy je funkci dodán konkrétní vstup, se nazývá **aplikace funkce**.
  - Příklad:

    ```
    plus 4 5
    ```
- Funkcionální programování
  - Program je soubor definic funkcí.
  - Tyto definice zachycují náš konkrétní problém.
  - Požadovaný výpočet pak je vyhodnocení *hlavního* výrazu.
    - Jak vyhodnotit výraz: *(7-3)*2* ?
    - Zde mám různé možnosti a strategie *(líne vyhodnocování...)*.

What is a function? More questions?

- *A box* that takes an *item* from functions domain and transforms it into an *item* from functions codomain.



- What is an item in the input domain or output codomain?

- How can we define a function? Which tools we need?
- What can we do with funcitons? Can we compose new functions from existing functions? Can function use other functions?
- How can we build a program from functions?

Input and output of functions

- Can we use more than one item in the input or output?
  - *From definition*: function takes $x \in X$ and *produces* exactly one $y \in Y$, it is usually denoted as: $f : X \to Y$.

$$f : \mathbb{N} \to \mathbb{N}$$
$$f(x) = x + 5$$

- But there are binary (n-ary) functions?
  - It is defined as: $f : X \times Y \to Z$
  - Element in Cartesian product is: $(x, y) \in X \times Y$

$$g : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$$
$$g(x, y) = x + y$$

- We can use another clever trick to introduce more arguments: *a function can return a function* (more details later).

$$h : \mathbb{N} \to (\mathbb{N} \to \mathbb{N})$$

How can we define a function?

- In computer science, a function is defined by:
  - is **type** signature;
  - and by *a rule*, that assigns an output value for an input value.
- Generally, in computer science, this rule can be a set of instructions (statements).
- In Haskell it is restricted to **expressions** (in fact same as mathematical expressions).
  - It is a *syntactic* entity that may be evaluated to determine its value.
  - It is a combination of one or more constants, variables, functions, and operators.
  - Expressions are evaluated according to its particular rules of precedence and of association (in Haskell, you can safely assume, that it is the same as in math).
- To create expressions we need:
  - basic elements - numbers, characters... $\rightarrow$ basic data types;
    - There is a type `Int` roughly representing natural numbers (set $\mathbb{N}$).
  - basic functions and operators;
    - There are binary operators like $+, -, *$ that works with the type `Int`.
    - Their type signature is: `Int -> Int -> Int`
  - some *mechanisms* to define more complex expressions.

How to build more complex functions?

- We need some syntactic constructs for *branching* → Nothing new, we know it from math!

$$abs(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{otherwise} \end{cases}$$

- A function can **use** another functions! → We can define hierarchy and reuse repeating patterns.

$$f(x, y) = abs(x) + abs(y) + abs(1) + 1$$

- Functions are first-class citizens in Haskell (as in math) → a function can be used as parameter or as a return value → it is a normal value.
  - From mathematics, we know function composition (denoted as $\circ$).
  - It is an *operator* (like $+$), that takes two functions f and g, and produces a function: $h = g \circ f$ such that: $h(x) = g(f(x))$.

$$test1(x) = (sin \circ abs)(x)$$
$$test2(f, x) = (f \circ abs)(x)$$

How can we build a program from functions?

- We decompose the problem into smaller parts.
- These parts are implemented as elementary functions.
- We have a mechanisms, how to combine these functions together.
- So, a whole program will be a set of functions.
- Finally, we need some starting point for our program → usually a function named **main**.
- Still, we are missing some key ideas:
  - Can we compose any two functions? → Are there some rules for working with functions?
  - Long running computation → Something like a cycle in C++ → recursive functions.
  - How to store real data? → User defined data types

Nástroje

- Haskell Platform
  - Glasgow Haskell Compiler (Interpreter)
  - Stack - package manager
  - Hoogle - API documentation
- Visual Studio Code
  - *Haskell*
  - Haskero
  - Haskell GHCi Debug Adapter Phoityne

GHCi

- Základní použití

```
>ghci
GHCi, version 8.6.5: http://www.haskell.org/ghc/  :? for help
Prelude>2*(3+5)
16
```

- Soubor obsahující uživatelské definice

  `>ghci example.hs`

- GHCi příkazy:

  `:edit|:e [file.hs]`
  `:load [file.hs]`
  `:reload`
  `:quit`
  `:?`

GHC

- Základní použití

```
main = do putStr "Your name:"
          name <- getStr
          putStr "Hello " ++ name
```

- Tradiční kompilátor:

```
>ghc example.hs
```

- Výsledek bude spustitelný soubor.
- Kde jsou funkce, co je to to `do`?
  - Monády - sekvence akcí obalená *čístými funkcemi*.
  - Blíže k *reálnému* programu v jazyce Haskell.

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Praktická ukázka

- Příprava pracovního prostředí
- Použití interpretu GHCi

Formát vstupního souboru

- Soubor s příponou **.hs** (tento budeme používat)

```
module Example where
-- Function computing sum of two numbers
sum x y = x + y
```

- Soubor s příponou **.lhs**

```
> module Example where

Function computing factorial
> f n = if n == 0 then 1 else n * f (n-1)
```

Modul Prelude

- Jako v jiných jazycích, zdrojové kódy jsou rozčleněny na oddělené *části*, nazvané moduly v jazyce Haskell (balíčky v Javě, namespace v C#).
- Modul se skládá z definic funkcí a uživatelem definovaných datových typů.

```
module Ant where ...
...
```

- Modul může být importován

```
module Example where
import Ant
...
```

- Speciální balíček defaultně importovaný: `Prelude`.
  - Některé názvy funkcí jsou *obsazené*.
  - Zrušení importu:

```
import Prelude hiding (max, min)
```

Jména

- **Identifikátory** - začínají písmenem, nasleduje sekvence znaků, číslic, podtržítek a jednoduchých úvozovek.

  `abc, h_e_l_l_o, hello', hello123, HeLLo, Hello`
- Jména použitá v definicích pro *hodnoty* začínají **malým** písmenem.
- Typy a konstruktory typů začínají **velkým** písmenem.

```
data Tree a  = Leaf a
             | Node (Tree a) (Tree a)

f::Int -> Int -> Int
f x y = x + y
```

Zarovnání I

- Odsazení je důležité! Definuje strukturu zdrojových kódů.
- Vnitřně jsou použity konstrukce se znaky `{ }` `;` (podobně jako v C++).
- Divné chyby se ';'

```
funny x = x +
          1
          ERROR ... : Syntax error in expression (unexpected ';').
```

- Základní pravidlo → **všechny konstrukce se stejným odsazením patří do stejného jmenného prostoru**.
- Vnitřní jmenný prostor vyžaduje větší odsazení.
- Dávejte si pozor, mezery nemusí být totéž co tabelátor.

Zarovnání II

- Příklady
  - ```
    maxsq x y
            | sqx > sqy = sqx
            | otherwise = sqy
            where
                    sqx = sq x
                    sqy = sq y
                    sq :: Int -> Int
                    sq z = z*z
    ```
  - ```
    maxsq x y
            | sq x > sq y = sq x
            | otherwise = sq y
                    where
                        sq x = x * x
    ```

Are there some rules for working with functions?

- Consider function:

$$f : \mathbb{R} \rightarrow \mathbb{R}$$
$$f(x) = \sqrt{log(x)}$$

  Is the definition OK?
- What options we have, if we want to work with this function?
  - Better specification of the function's domain $\rightarrow$ In terms of computer science, introduce a new type like: $X = \{x \in \mathbb{R}, x \geq 1\}$, then $f : X \rightarrow \mathbb{R}$
  - Be *really careful* when evaluation this function $\rightarrow$ while evaluating, we will handle possible exceptional situations.
  - Do not worry at all (programmer is always right, what will be will be...:-)

Typový systém

- *Velmi důležitá součást každého programovacího jazyka..*
- **Neformální definice**
  - Typový systém asociuje jeden (nebo více) typ(ů) s každou hodnotou v programu.
  - Ohodnocením toku těchto hodnot se typový systém snaží dokázat, že nemůže nastat žádná "typová chyba".
- Přiřazení datových typů (typing) dává význam kolekci bitů.
- Typy jsou obvykle asociovány buď s hodnotami nebo s objekty, jako jsou promněnné.
- Typy umožňují programátorovi přemýšlet o programech na vyšší úrovni než bity a bajty, odprostí ho do nízko-úrovňové implementace.
- Použití typů může umožnit kompilátoru detekovat bezvýznamný, nebo nefunkční kód.
  - Toto je pravda zejména pro čistě funkcionální jazyky, kde obtížné *vyrobit chybu* v typově korektním programu.

Basic functionality of a type system (in Haskell) I

- **Haskell** has static, strong and safe type system. Moreover, it supports polymorphism.
  - Static typing *(C, C++, Java, Haskell...)*
    - Type checking is performed (mostly) during compile-time.
    - Static typing is a limited form of program verification.
    - It allows many errors to be caught early in the development cycle.
    - Static type checkers are conservative - they will reject some programs that may be well-behaved at run-time, but that cannot be statically determined to be well-typed.
  - Dynamic typing *(Javascript, Python, PHP...)*
    - Majority of its type checking is performed at run-time.
    - Dynamic typing can be more flexible than static typing. For example by allowing programs to generate types based on run-time data.
    - Run-time checks can potentially be more sophisticated, since they can use dynamic information as well as any information that was present during compilation.

```
var x := 5;      // (1)  (x is an integer)
var y := "37";   // (2)  (y is a string)
var z := x + y;  // (3)  (? - Visual Basic = 42, Javascript "537")
```

Basic functionality of a type system (in Haskell) II

- Strongly typed languages - do not allow undefined operations to occur.
- Weak typing means that a language implicitly converts (or casts) types when used.
- Type safe - is language if it does not allow operations or conversions which lead to erroneous conditions.
- Memory safe - for example it will check array bounds (resulting to compile-time and perhaps run-time errors).

```
int x = 5;
char y[] = "37";
char* z = x + y; //z points five characters after y
```

- Polymorphism
  - The ability of code (in particular functions, methods or classes) to act on values of multiple types, or the ability of different instances of the same data-structure to contain elements of different types.
  - Type systems that allow polymorphism generally do so in order to improve the potential for code re-use.

Basic functionality of a type system (in Haskell) III

```
Animal obj = new Horse();
obj.sound();
```

```
length :: [a] -> Int -- a is a type variable
length [] = 0
length (x:xs) = 1 + length xs
```

- **Type checking** - the process of verifying and enforcing the constraints of types.
- **Type interference**
  - Strongly statically typed languages
  - Automatic deduction of the data types
  - Hindley-Milner type system

Basic data types I

- `1::Int`

  `+, -, *, ^, div, mod, abs, negate, ==`
- `'a'::Char`
  - Special characters: `'\t'`, `'\n'`, `'\\'`,`'\''`, `'\"'`
  - Prelude functions:

    `ord :: Char -> Int, chr :: Int -> Char, toUpper, isDigit`
  - Library `Char`
- `True`,`False::Bool`

  `&&, ||, not, ==`
- `3.14::Double` (`3.14::Float`)

Basic data types II

```
+, -, *, /, ^, **,
==, /=, <, >, <=, >=
abs, acos, asin, sin, cos,
celing, floor, exp, fromInt, log, negate, pi
```

- `"Hello"::String`
  - Defined as: `type String = [Char]` - list of characters, lists will be explained later.
  - Example: `"Hello world"`

Summary, how to work with these basic data types

- Each type is defined by its *unique* name (starting with capital letter in Haskell) - `String`.
- There are some build-in types, later we will learn a way to build our own types.
- There is a way, how to write a constant value (literal) of these types.
- *Values* belongs to exactly one type.
- There are basic functions and operators working with these basic data types.
- If you want to define a type (for a value or a function), you can use: `1::Int`

Function's definition and its type

- Definition

```
name :: Type
name parameters = expression
```

- Example

```
square :: Int -> Int  -- optional!
square n = n * n

sum :: Int -> Int -> Int
sum x y = x + y
```

- Function application

```
square 5 = 5 * 5
square (2+4) = (2+4) * (2+4)
sum 4 5
```

So, how to really write a function in Haskell?

$$f : \mathbb{N} \to \mathbb{N}$$
$$f(x) = x + 5$$

```haskell
f :: Int -> Int
f x = x + 5
```

$$g : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$$
$$g(x, y) = x + y$$

```haskell
g :: (Int, Int) -> Int
g (x,y) = x + y
```

- But on previous slide, there was: `sum :: Int -> Int -> Int`
- *Real way*, how to write binary function in Haskell.

So, how to really write a function in Haskell?

- What is: `->`
  - In fact, it is *an operator* with **right** associativity.
  - So, `sum :: Int -> Int -> Int` is equivalent to `sum :: Int -> (Int -> Int)`.
  - How can we understand it? $\rightarrow$ Remember, functions are also values $\rightarrow$ It is a function that returns a function!
- Functional programming languages are *based* on Lambda calculus.
  - Lambda abstraction $\Leftrightarrow$ anonymous function: $\lambda x.x + 5$
  - `sum x y = x + y` $\Leftrightarrow \lambda x.(\lambda y.x + y) \Leftrightarrow \lambda xy.x + y$
- What can we do with such lambda expressions? $\rightarrow$ We can apply them on some values.
  - How it is evaluated? $\rightarrow$ We *substitute* the corresponding variable with the value.
  - $(\lambda x.x + 5 \quad 2) \rightarrow (x + 5)[x := 2] \rightarrow 2 + 5 \rightarrow 7$
  - $((\lambda x.(\lambda y.x + y) \quad 2) \quad 3) \rightarrow ((\lambda y.x + y)[x := 2] \quad 3) \rightarrow (\lambda y.2 + y \quad 3) \rightarrow (2 + y)[y := 3] \rightarrow 2 + 3 \rightarrow 5$
- Functions application is left associative.
  - `((sum 2) 3)` $\Leftrightarrow$ `sum 2 3`

So, how to really write a function in Haskell?

- Where are we now?
  - We can write a function using the basic data types.
  - The function will be using basic operators and functions associated with these types.
  - We know, how to create a *n-ary* function and how to use it.
- What are the most fancy things we can do now?

```
applyTwice :: (Double -> Double) -> Double -> Double
applyTwice f x  = f (f x) -- applyTwice sin 1
```

  - What if we want to use it with `Int` and function `negate`?
  - We talked about polymorphism → We can use parameters (variables) in type definition.

```
applyTwice :: (a -> a) -> a -> a
applyTwice f x  = f (f x) -- applyTwice sin 1, applyTwice negate 1
```

  - Still, we need to be careful, types needs to be valid.

So, how to really write a function in Haskell?

- Can we use more parameters in type definition?
- Function composition: $h = g \circ f$ such that: $h(x) = g(f(x))$.

```
compose :: (b -> c) -> (a -> b) -> (a -> c)
compose g f = -- ???
```

- We still do not have tools to build more complex functions...

So, how to really write a function in Haskell?

- Can we use more parameters in type definition?
- Function composition: $h = g \circ f$ such that: $h(x) = g(f(x))$.

```
compose :: (b -> c) -> (a -> b) -> a -> c
compose g f x = g (f x)
```

- We still do not have tools to build more complex functions...

Interlude - Type classes - what does the => mean?

- Assume the following function sum, what is its type?

```
sum x y = x + y
```

```
Prelude> :type sum
sum :: Num a => a -> a -> a
```

- `Num` is a type class for all numbers, a is a type variable. It can be any *numeric* type, for example `Double` or `Int`.

```
Prelude> :type (sum (4::Int) 4)
(sum (4::Int) 4) :: Int
Prelude> sum 1.1 1
```

- We can restrict the function type.

```
sum :: Int -> Int -> Int
sum x y = x + y
```

Practical demonstration

- Identifiers in Haskell
- What is a Haskell program?
- Primitive data types.
- How to write an expression.
- Calling some standard functions.
- Using some operators for primitive data types.
- Checking types of these functions.
  - What is the meaning of *the nonsense* that operator + returns as its type?

Syntax of function's definition

- **Pattern matching**
- Guard expressions

```
max :: Int -> Int -> Int
max x y | x>=y = x
        | otherwise = y
```

- Local definitions - `where`
  - Local definitions need to have *bigger indentation*.
  -
```
initials :: String -> String -> String
initials firstname lastname = [f] ++ ". " ++ [l] ++ "."
    where f = head firstname
          l = head lastname
```

Pattern matching

- Several function definitions (equations) with different *patterns*.

```
f  pat11  pat12  ...  = rhs1
f  pat21  pat22  ...  = rhs2
...
```

- First equation that can be unified with given parameters is chosen.

```
f  value1 value2 ...
```

- If there is none → **error**
- The most basic patterns are:
  - **constants**;
  - **variables**.

Pattern matching - naive example

- Basic example:

```
sayMe 1 = "One!"
sayMe 2 = "Two!"
```

- Some function applications:

```
*Main> sayMe 1
"One!"
*Main> sayMe 3
"*** Exception: input.hs:(1,1)-(2,16): Non-exhaustive patterns in
function sayMe
```

- First equation that can be unified with given parameters is chosen.

```
sayMe 1 = "One!"
sayMe 2 = "Two!"
sayMe x = "Something else"
```

So, how to really write a function in Haskell?

Pattern matching - better example

- The answer to most questions in Haskell is recursion.
- Recursive function is a function that *calls* itself.
  - Very nice is a *tail recursion*..
- Simple example of recursion is **factorial** in math.
  $n! = n \times (n-1) \times (n-2) \times (n-3) \times \cdots \times 3 \times 2 \times 1$
- How to write it in Haskell?

```
factorial :: Int -> Int
factorial 0 = 1
factorial 1 = 1
factorial n = n * factorial (n-1)
```

- Pattern matching is a syntactic sugar based on case expression.

Interlude - operators and priority

- What about the following expression, how to understand it? Which brackets are necessary?

  `n * factorial (n-1)`
- In Haskell, like in other languages (C++), there are functions and operators.
- Operators are composed from characters:

  `! # & $ % * + - . / < > = ? \ ^ | : ~`
- Operators are using *infix* notation (`5 + 3`).
- Priority rules:
  - Function application has a highest priority.
  - Operator `*` have a higher priority then `+`.
- Operators and their priority will be explained later. **If not sure use brackets!**

Pattern matching - application

- Factorial definition

```
factorial 0 = 1
factorial 1 = 1
factorial n = n * factorial (n-1)
```

- Functions application step by step.

```
factorial 5 = 5 * factorial (5-1) = 5 * factorial 4
            = 5 * 4 * factorial 3
            = 5 * 4 * 3 * factorial 2
            = 5 * 4 * 3 * 2 * factorial 1
            = 5 * 4 * 3 * 2 * 1
            = 120
```

So, how to really write a function in Haskell?

Syntax for expressions

- Expressions can be used **anywhere**!
- We already know expressions - function's application and a usage of operator.
- `if` *expression* - it is similar to ternary `?:` operator from C++ `((x>y)? x : y)`

```
max x y = if x > y then x else y
```

- `case` expression

```
describeList :: [a] -> String
describeList xs = "The list is " ++ case xs of
        [] -> "empty."
        [x] -> "a singleton list."
        xs -> "a longer list."
```

- `let` expression

```
cylinder r h = let sideArea = 2 * pi * r * h
                   topArea = pi * r ^2
               in  sideArea + 2 * topArea
```

Practical demonstration

- Priority in expressions.
- Implementing some simple functions.
- Defining function's type.
- Type inheritance.
- Where can I store some data if needed? `-- How to declare variables?`
- How can I write *a cycle*? `-- I NEED my cycles!`

Lists

- Probably the most used data structure in functional Languages (for C++ the equivalent will be array).
- Lists are a **homogeneous** data structures.
  - A list can contain only elements with the same data types.
  - `[1,2,3] -- OK`
  - `[1,'a',3] -- Wrong`
  - `"hello" == ['h','e','l','l','o']`
- Informally, the term **syntactic sugar** refers to **a nicer way** how to write something.

List - Basic concept

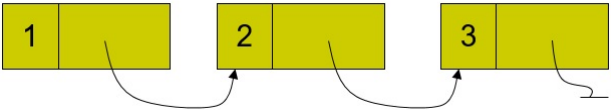

Figure: A scheme of the list.

- Element in a list contains the data and a reference to other elements.
- The last element points *nowhere* (usually a sort of **null** reference).

- How to work with such a list?
  - What if i want to get n^{th} element in the list?

List - Basic concept

	Array	List	Winner
Get n^{th} element	pointer arithmetic's	need to go trough the list	Array
Add element at the beginning	new memory, copy all	easy, just add	List
Add element at other positions	new memory, copy all	rebuild the list	Tie
Remove element at the beginning	new memory, copy relevant	easy, get second element	List
Remove element at other positions	new memory, copy relevant	rebuild the list	Tie
Modify first element	get and modify	new first, stitch the tail	List
Modify any other element	get and modify	rebuild the list	Array

Table: Informal comparison of an array and a list

Usage of lists usually requires different algorithms.

Lists in Haskell

- Definition

```
data List a = Cons a (List a)
            |  Nil
```

- Application of (syntactic) sugar
  - `List a` → `[a]`
  - `Cons a` → `: -- a -> [a] -> [a]`
  - `Nil` → `[]`
- List literals
  - `[1,2,3] :: [Int]`
  - `1:2:3:[] :: [Int]`
- Patterns for lists
  - Empty list `[]`
  - Non-empty list `(x:xs)`

Function working with lists

- List `length` function

```
length [] = 0
length (x:xs) = 1 + length xs
```

- Application of this function

```
length [1,2,3] = 1 + length [2,3]
               = 1 + 1 + length [3]
               = 1 + 1 + 1 + length []
               = 1 + 1 + 1 + 0 = 3
```

- What is the type of this functions?
  - Do we even know or care about the type of the element in the list?

```
length :: [a] -> Int
length [] = 0
length (_:xs) = 1 + length xs
```

Tuples

- Haskell have build in ordered tuples `(a,b,c,d,...)`

```
(1,2) :: (Int,Int)
(1,['a','b'], "abc")::(Int, [Char], String)
() :: ()
```

- Unlike homogeneous lists, tuples can have elements of different types.
- Example of a *pattern* for tuples:

```
addThem :: (Int, Int) -> Int
addThem (x,y) = x + y
```

- Build in functions working with tuples.

```
addThem :: (Int, Int) -> Int
addThem x = fst x + snd y
```

Type classes 101 (more details later)

- Not the same as classes form Java or C++.
- Type of the operator ==

```
ghci> :t (==)
(==) :: (Eq a) => a -> a -> Bool
```

- Definition of the type class Eq

```
class Eq a where
  (==), (/=) :: a -> a -> Bool
  x /= y =  not (x == y)
```

- A type can become a member of this class, if it provides functions and operators that the class defines.

Practical demonstration

- `List`
  - Basic type storing *bigger* data - `List -- Who needs arrays...`
  - Functions for *all* lists (type variables → polymorphism).
  - Simple functions *going trough* the list.
  - Nice patterns available to handle lists.
- Tuples

Basic type classes

- Class `Eq` - `== /=`
- Class `Ord` - `> < >= <= compare`
- Class `Show` - `show :: a -> String`
- Class `Read` - `read :: (Read a) => String -> a`
  - Why is it not working?

```
ghci> read "4"
<interactive>:1:0:
    Ambiguous type variable `a' in the constraint:
      `Read a' arising from a use of `read' at <interactive>:1:0-7
    Probable fix: add a type signature that fixes these type variable(s)
```

  - We can repair it by: `read "4" :: Int`
- Class `Enum` - `succ, pred`
- Class `Bounded` - `minBound, maxBound :: (Bounded a) => a`

Basic type classes for numbers

- Basic relations between numeric classes `Num` (not all numeric classes are mentioned)
  - `Num` → `Real`, `Fractional`
  - `Real` → `Integral`, `RealFrac`
  - `Fractional` → `RealFrac`, `Floating`

  - `Integral` → `Int`, `Integer`
  - `Floating` → `Float`, `Double`

Conversion between numbers

- There are functions taking a value and pushing it *higher* in the type hierarchy.

```
fromIntegral :: (Num b, Integral a) => a -> b

fromIntegral (length [1,2,3,4]) + 3.2
```

- There are functions changing the type class to a class in the same *level*

```
realToFrac :: (Real a, Fractional b) => a -> b
```

- Special functions

```
round :: (RealFrac a, Integral b) => a -> b
```

- There is a lot of functions converting types of numeric values.

```
fromInteger, toInteger, fromRational, toRational, ceiling,
floor, truncate
```

Partially applied functions

- In theory, every Haskell function only takes one parameter.
  - But we were using functions with several parameters? → **curried functions**
  - Definition `max :: (Ord a) => a -> a -> a` can be rewritten as: `max :: (Ord a) => a -> (a -> a)`.
- What really happens when a function is applied? → `(max 2) 3`
  - It will work even if we specify just one parameter `max 2` → **partially applied function**

```
Prelude> :t max 2
max 2 :: (Ord a, Num a) => a -> a
```

- **Functions can return other functions**

```
compareWithHundred :: (Num a, Ord a) => a -> Ordering
-- compareWithHundred x = compare 100 x
compareWithHundred = compare 100
```

High order functions

- Functions can have other functions as parameters.

```
applyTwice :: (a -> a) -> a -> a
applyTwice f x = f (f x)
```

- Useful example of a high order function.

```
map :: (a -> b) -> [a] -> [b]
map _ [] = []
map f (x:xs) = f x : map f xs
```

- How it is used?

```
map fst [(1,2),(3,5),(6,3),(2,6),(2,5)]  -- [1,3,6,2,2]
```

Practical demonstration

- What is a type class?
- Useful type classes.
- Function is a first class citizen in Haskell.
  - Partial application, curried functions.
  - Usage of high order functions.
- Some tips how to write complex functions.
  - Dividing complex computations into smaller functions.
  - Construct `let ... in`

Operators in Haskell

- Operators are composed from characters: `!#&$%*+-./<>=?\^:|`
- Operators are using *infix* notation (`5 + 3`).
- Important for operators are priority and *associativity*.
- Operators can be used as functions.

  ```
  (+) 1 2
  ```

- Functions can be use as operators.

  ```
  5 `mod` 3
  ```

- This change affects also the priority!
  - We can define the priority of an operator created from a function.

    ```
    infixl 7 `mod`
    ```

Standard operators

Precedence	Operator	Description	Associativity
9	.	Function composition	Right
8	^,^^,**	Power	Right
7	*,/,`quot`,`rem`,`div`,`mod`		Left
6	+,-		Left
5	:	Append to list	Right
4	==,/=,<,<=,>=,>	Compare-operators	
3	&&	Logical AND	Right
2	||	Logical OR	Right
1	>>,>>=,=<<		
0	$,$!,`seq`		Right

Creating new operators

- Operators are defined similarly to functions.

```
(&&&) :: Int -> Int-> Int
x &&& y = x + y
```

- We can change the precedence and associativity.

```
infixl 6 &&&
```

- Associativity can be changed by: `infix`, `infixl`, `infixr`
  - Keyword `infix` defines *no associativity*. We need explicit parenthesis.

Numeric lists

- `[m..n]`

  ```
  [1..5] -- [1,2,3,4,5]
  ```

- `[m1,m2..n]`

  ```
  [1,3..10] -- [1,3,5,7,9]
  ```

- Never-ending list - `[m..]`

  ```
  [1..] -- [1,2,3,4,5,...]
  ```

- `[m1,m2..]`

  ```
  [5,10..] -- [5,10,15,20,25,...]
  ```

List comprehension

- Consider mathematical definition → Define a set containing even natural numbers smaller then or equal to 10.

```
[n | n <- [1..10], n `mod` 2 == 0]
```

- Examples

```
[x*2 | x <- [1..10]]  -- [2,4,6,8,10,12,14,16,18,20]
[x*2 | x <- [1..10], x*2 >= 12] -- [12,14,16,18,20]
[ x*y | x <- [2,5,10], y <- [8,10,11]] -- [16,20,22,40,50,55,80,100,110]
allEven xs = xs == [x | x<-xs, isEven x]
allOdd xs = xs == [x | x<-xs, not(isEven x]
length' xs = sum [1 | _ <- xs]
```

Never-ending (infinite) lists

- Can not *show* the list `[1..]` but we can still use it *(lazy evaluation)*.
- Consider following function **zip**.

```
zip :: [a] -> [b] -> [(a, b)]
zip [] _ = []
zip _ [] = []
zip (x:xs) (y:ys) = (x,y) : zip xs ys

zip [1,2,3] "ABCD" -- [(1,'A'),(2,'B'),(3,'C')]
zip [1..] "ABCD" -- [(1,'A'),(2,'B'),(3,'C'),(4,'D')]
```

Practical demonstration

- Defining new operators.
- List comprehensions:
  - it can simplify the solution;
  - nice examples of its usages.
- Nice examples of usages of infinite lists. `--Are they even USEFUL?`
- Lambda expressions.

Basic functions for lists I

- Accessing list elements

```
head [5,4,3,2,1] -- 5
tail [5,4,3,2,1] -- [4,3,2,1]
last [5,4,3,2,1] -- 1
init [5,4,3,2,1] -- [5,4,3,2]
[1,2,3] !! 2  -- 3
length [5,4,3,2,1] -- 5
null [1,2,3] -- False
null [] -- True
```

- Merging lists

Basic functions for lists II

```
[1,2,3] ++ [4,5] -- [1,2,3,4,5]
concat [[1,2],[3],[4,5]] -- [1,2,3,4,5]
zip [1,2] [3,4,5] -- [(1,3),(2,4)]
zipWith (+) [1,2] [3,4] -- [4,6]
```

- Computing with lists

```
minimum [8,4,2,1,5,6] -- 1
maximum [1,9,2,3,4] -- 9
sum [5,2,1,6,3,2,5,7] -- 31
product [6,2,1,2] -- 24
```

- Taking a part of a list

Basic functions for lists III

```
take 3 [5,4,3,2,1] -- [5,4,3]
drop 3 [8,4,2,1,5,6] -- [1,5,6]
takeWhile (> 0) [1,3,0,4] -- [1,3]
dropWhile (> 0) [1,3,0,4] -- [0,4]
filter (> 0) [1,3,0,2,-1] -- [1,3,2]
```

- Transforming a list

```
reverse [5,4,3,2,1] -- [1,2,3,4,5]
map (*2) [1,2,3] -- [2,4,6]
```

- Selected *nice* functions

```
4 `elem` [3,4,5,6] -- True
replicate 3 10 -- [10,10,10]
-- cycle and repeat returns infinite  list
take 10 (cycle [1,2,3]) -- [1,2,3,1,2,3,1,2,3,1]
take 10 (repeat 5) -- [5,5,5,5,5,5,5,5,5,5]
```

Folding a list I

- In general, folding functions transform `Foldable` structure to a *value*.
- `Foldable` structure can be for example a tree → for such a structure we need to define how to traverse it.
- We will be using it only for lists → lists are *foldable* structures.

```
class Foldable (t :: * -> *) where
  foldl :: (b -> a -> b) -> b -> t a -> b
  foldr :: (a -> b -> b) -> b -> t a -> b
  foldr1 :: (a -> a -> a) -> t a -> a
  foldl1 :: (a -> a -> a) -> t a -> a
```

- Examples

Folding a list II

```
sum' :: (Num a) => [a] -> a
sum' x = foldl (+) 0 x

product' :: (Num a) => [a] -> a
product' x = foldr1 (*) x
```

- Functions `scanl`, `scanr`, `scanl1`, `scanr1` are like their *fold* counterparts, only they report all the intermediate accumulator states in the form of a list.

```
scanl (+) 0 [3,5,2,1] -- [0,3,8,10,11]
scanr (+) 0 [3,5,2,1] -- [11,8,3,1,0]
```

Lambda expressions

- Lambdas are basically anonymous functions.
  - They are used only once → so they do not need even a name.
- Syntax

```
\x y -> x + y
```

- Examples

```
map (\(a,b) -> a + b) [(1,2),(3,5),(6,3),(2,6),(2,5)] -- [3,8,9,8,7]

reverse' :: [a] -> [a]
reverse' = foldl (\acc x -> x : acc) []

elem' :: (Eq a) => a -> [a] -> Bool
elem' y ys = foldl (\acc x -> if x == y then True else acc) False ys
```

Function application with $

- Definition:

```
($) :: (a -> b) -> a -> b
f $ x = f x
```

- Differences with function application.
  - Function application is left-associative (`((f a) b) c`), $ right-associative.
  - Function application have has a highest precedence, $ has the lowest precedence.
- Why it is useful? → It eliminates many parentheses.

```
sum (map sqrt [1..130])  = sum $ map sqrt [1..130]
sqrt (3 + 4 + 9) = sqrt $ 3 + 4 + 9
```

- It also means, that function application can be treated just like another function!

```
ghci> map ($ 3) [(4+), (10*), sqrt]
```

Function composition

- Definition:

```
(.) :: (b -> c) -> (a -> b) -> a -> c
f . g = \x -> f (g x)
```

- The meaning is the same as in math - compose a function that takes the input, applies g and then on the result f.
- It is right-associative and it has a high precedence.

```
(\x -> negate (abs x)) = (negate . abs)
fn = ceiling . negate . tan . cos . max 50
```

User defined data types - introduction

- Type synonyms (preserve type classes)

```
type String = [Char]
type Table a = [(String, a)]
type AssocList k v = [(k,v)]
```

- New (algebraic) data type

```
data Bool = False | True
data Color  = Black | White | Red

isBlack :: Color -> Bool
isBlack Black = True
isBlack _ = False
```

  - Color – type constructor
  - Red / Green / Blue – data (nullary) constructor

User defined data types - more advanced I

- Parametric data types

```
data Point = Point Float Float
```

- Data (Value) constructor's type

```
ghci> :t Point
Point :: Float -> Float -> Point
```

- Usage

```
dist (Point x1 y1) (Point x2 y2) = sqrt ((x2-x1)**2 + (y2-y1)**2)
```

```
ghci> dist (Point 1.0 2.0) (Point 4.0 5.0) = 5.0
```

- Polymorphic data types

User defined data types - more advanced II

```
data Point a = Point a a
```

- Constructor: `Point :: a -> a -> Point a`
- Better examples (build in types)

```
data Maybe a = Nothing | Just a
data Either a b = Left a | Right b deriving (Eq, Ord, Read, Show)

sqrt' :: Float -> Maybe Float
sqrt' x | x < 0     = Nothing
        | otherwise = Just (sqrt x)
```

Recursive data types

- We already know recursive data type - List

```
data List a = Null
            | Cons a (List a)

lst :: List Int
lst = Cons 1 (Cons 2 (Cons 3 Null))
```

- Better example - binary tree

```
data Tree1 a = Leaf a | Branch (Tree1 a) (Tree1 a)
data Tree2 a = Leaf a | Branch a (Tree2 a) (Tree2 a)
data Tree3 a = Null | Branch a (Tree3 a) (Tree3 a)

t2l :: Tree1 a -> [a]
t2l (Leaf x) = [x]
t2l (Branch lt rt) = (t2l lt) ++ (t2l rt)
```

Automatically deriving type classes

- Consider following example:

```
data Color = Black | White
list :: [Color]
list = [Black, Black, White]
```

```
ghci> list
<interactive>:15:1: error:
    * No instance for (Show Color) arising from a use of `print'
    * In a stmt of an interactive GHCi command: print it
```

- A solution can be let Haskell automatically derive type classes.

```
data Color = Black | White deriving (Show, Eq, Ord, Read)
```

```
ghci> list
[Black,Black,White]
```

Record syntax

- Named fields in a data type definition.

```
data Person = Person { firstName :: String
                     , lastName :: String
                     , age :: Int
                     } deriving (Show)
```

```
ghci> :t firstName
firstName :: Person -> String
```

```
description :: Person -> String
description p = firstName p ++ " " ++ lastName p
```

```
description Person {firstName = "John" , lastName="Doe", age = 40}
```

Practical demonstration

- New user defined data types.
  - Algebraic types
  - Parametric data types
  - Polymorphic data types
  - Simple recursive data structures (list).
- Data structures handling frequently encountered problems.
  - Maybe
  - Either
  - Tree - Expressions
- Record syntax

Modules 101

- Module definition
  - All definitions are visible.

    ```
    module A where   -- A.hs, A.lhs
    ```
  - Restricted export

    ```
    module Expr ( printExpr, Expr(..) ) where
    ```
  - Data types restrictions

    ```
    Expr(..) -- exports also constructors
    Expr -- exports data type name only
    ```
- Restricted import

  ```
  import Expr hiding( printExpr )
  import qualified Expr -- Expr.printExpr
  import Expr as Expression -- Expression.printExpr
  ```

Type classes 102 I

- Type class definition

```
class Eq a where
    (==) :: a -> a -> Bool
    (/=) :: a -> a -> Bool
    x == y = not (x /= y)
    x /= y = not (x == y)
```

  - Default definitions are overridden by instance definition.
  - At least one must be defined.

- Adding a type into a class.

```
instance Eq Color where
    Black == Black = True
    White == White = True
    _ == _ = False
```

Type classes 102 II

- Adding a data type with parameters.

```
instance (Eq a) => Eq (Maybe a) where
    Just x == Just y = x == y
    Nothing == Nothing = True
    _ == _ = False
```

- There can be relations between data type classes - class `Ord` inherits the operations from class `Eq`.

```
class Eq a => Ord a where
        (<), (<=), (>), (>=) :: a -> a -> Bool
        max, min :: a -> a -> a
        compare          :: a -> a -> Ordering
```

Example of a user defined type class I

```
class Visible a where
        toString :: a -> String
        size :: a -> Int
instance Visible Char where
        toString ch        = [ch]
        size _ = 1
instance Visible Bool where
        toString True = "True"
        toString False = "False"
        size = length . toString
```

Example of a user defined type class II

```
instance Visible a => Visible [a] where
        toString          = concat . map toString
        size          = foldr (+) 0 . map size

class (Ord a, Visible a) => OrdVisible a where
    ...
```

Abstract data type

- Definition: An abstract data type is defined as a mathematical model of the data objects that make up a data type as well as the functions that operate on these objects.
- Imperative style
  - Class of objects whose logical behavior is defined by a set of values and a set of operations.
  - ADT is a *mutable* structure (mutable structure has inner state, its behaviour can change in time).
- Functional style
  - Each state of the structure as a separate entity.
  - ADT is modeled as a mathematical function that takes the old state as an argument, and returns the new state as part of the result.
  - Unlike the imperative operations, these functions have no side effects.

Example of queue usage in Java

```
public class QueueExample
{
  public static void main(String[] args)
  {
    Queue<Integer> q = new LinkedList<>();

    for (int i=0; i<5; i++)
     q.add(i);

    int remove = q.remove();
    int head = q.peek();
    int size = q.size();
  }
}
```

Queue implementation in Haskell I

- Initialization: `emptyQ :: Queue a`
- Test if queue is empty: `isEmptyQ :: Queue a -> Bool`
- Inserting new element at the end: `addQ :: a -> Queue a -> Queue a`
- Removing element from the beginning: `remQ :: Queue q -> (a, Queue a)`

```
module Queue(Queue, emptyQ, isEmptyQ, addQ, remQ) where
    data Queue a = Qu [a]

    emptyQ :: Queue a
    emptyQ = Qu []

    isEmptyQ :: Queue a -> Bool
    isEmptyQ (Qu q) = null q
```

Queue implementation in Haskell II

```
addQ :: a -> Queue a -> Queue a
addQ x (Qu xs) = Qu (xs++[x])

remQ :: Queue a -> (a,Queue a)
remQ q@(Qu xs) | not (isEmptyQ q) = (head xs, Qu (tail xs))
               | otherwise        = error "remQ"
```

Practical demonstration

- Functional style for handling data:
  - defining new data types;
  - how to handle these new data;
  - modules. `--Just to avoid long files?`

Thank you for your attention

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VSB TECHNICAL UNIVERSITY OF OSTRAVA | FACULTY OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE | DEPARTMENT OF COMPUTER SCIENCE